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ON SOUND INTENSITY AND SOUND PRESSURE LEVELS

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TECHNICAL MEMORANDUM X-53035

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By

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ABSTRACT

Sound propagation through the open atmosphere is studied at MSFC mainly for an estimate of the acoustical energy that sound rays sent up by static firings and refracted back to ground level may transmit to inhabited areas. A theoretical expression derived in an earlier report (Ref. 1) for the volume density of returned energy is converted into an expression for the corresponding intensity level to accommodate it to engineering practice. A first approximation of the latter's relationship to the sound pressure level (as an observable quantity) is established. The results of the theory can thus be compared to those of field measurements by microphones, and a basis for theoretical prediction is prepared.

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DEFINITION OF SYMBOLS

Symbol	Definition
p	oscillatory pressure
ρ	density
s	specific entrophy
$\underline{w}$	velocity vector in a paradigmatic sinusoidal sound wave of circular frequency, ω
(o)	superscript referring to conditions in the undisturbed atmosphere
$\underline{w}^{(o)}$	wind vector
$\sim$	symbol used to denote the overall state in the vibrating atmosphere, e.g., $\tilde{p} = p^{(o)} + p$, $\tilde{\underline{w}} = \underline{w}^{(o)} + \underline{w}$
$\wedge$	symbol used to designate amplitude, e.g., $p = \hat{p}e^{(\omega t - k_o W)i}$, $\underline{w} = \hat{w}e^{(\omega t - k_o W)i}$
W	spatial coordinate
k_o	wave number
T	time required for full oscillation
ϵ	volumn density of sound energy
$\underline{V}_s$	directed propagation speed of sound energy
$I = \epsilon\lvert\underline{V}_s\rvert$	sound intensity

DEFINITION OF SYMBOLS (Continued)

Symbol	Definition
I^*	reference intensity (either one of two values, I_1^* and I_2^*, are commonly used - see p. 4)
$\underline{n}$	unit vector in the direction of the local wave front normal
V_f	wave speed in this direction

DEFINITIONS

SOUND PRESSURE as recorded by instruments is the square root of the time average of p^2:

$$p_m = \sqrt{\frac{1}{T}\int_0^T p^2\,dt} = \frac{1}{2}\hat{p}^2 .$$

REFERENCE PRESSURE: $p^* = 0.0002$ microbar.

SOUND PRESSURE LEVEL: $20 \log_{10} \frac{p_m}{p^*}$.

INTENSITY LEVEL: $10 \log_{10} \frac{I}{I^*}$.

P: power of the sound source. The reference value for the power is taken as 10^{-13} watt, so that POWER LEVEL is given by:

$$D = 10 \log \frac{P}{10^{-13}} .$$

A fuller mode of writing the left side indicates the reference value: D db re 10^{-13} watt. The unit for levels is always the decibel.

$c^{(o)} = \sqrt{\gamma \frac{p^{(o)}}{\rho^{(o)}}}$: thermodynamic sound speed in the undisturbed atmosphere.

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SUMMARY

An expression is given for the rearrival intensity level of sound emitted by a point source, if an infinitesimal ray bundle, without experiencing physical attenuation of energy, is returned to the horizontal plane on which the source is sitting. Its relation to the sound pressure level, so far known only in the case of plane and spherical waves, has been set up in general using a first approximation approach. It was found that the sound pressure level at ground is, in many cases, not greatly different from the intensity level, and that the difference can be easily computed. A theoretical value of the sound pressure level is thus made available which may be compared to the sound pressure level as determined experimentally at any given return distance.

I. THEORETICAL INTENSITY OF RETURNING SOUND RAYS

Common practice in calculating sound ray patterns assumes that rays which leave the point source in a vertical half-plane do not swerve from this plane later on so that a two-dimensional treatment becomes possible. Bundles of these rays may return to the ground at source level. If the plane can be divided up into horizontal layers in which the propagation velocity of sound varies linearly with height alone, it is possible to give an explicit formula for the landing distance, x_s, of such rays which, aside from fixed parameters, depends on the initial angle, θ_o, of ray elevation:

$$x_s = f(\theta_o).$$

In Reference 1, the volume density, ϵ_s, of the acoustical energy transmitted to ground level by an infinitesimally small ray bundle, with nearly equal angles of departure around θ_o, is given as

$$\epsilon_s = \bar{\epsilon}\,\frac{\bar{r}^2}{x_s}\left|\frac{d\theta_o}{dx_s}\right| \text{ cotg } \theta_o, \tag{1}$$

where $\bar{\epsilon}$ is the volume density of energy found on the surface of a sphere of sufficiently small radius $\bar{r}$ about the source.* Strictly speaking, the value of $\bar{\epsilon}$ ought to be determined experimentally; however, if the acoustical power of the source is known, the product $\bar{\epsilon}\bar{r}^2$ can be calculated from it to some degree of accuracy. In deriving the expression (1), physical attenuation of energy is not taken into account (while it includes the effects of geometrical ray spreading or compressing). The numerical value found for ϵ_s is therefore an upper bound rarely or never obtained in practice.

Field measurements of returned sound energy are usually quoted in terms of sound pressure levels. It is desirable to establish a correlation between the observational results and the theoretical prediction (1).

Since acoustical rays are defined as the propagation lines of energy, the volume element dA ds of a ray tube can be assigned the energy density ϵ, so that

$$dE = \epsilon \, dA \, ds$$

is the mean vibrational energy contained in the element. If V is the propagation velocity at the location considered, the arc length element ds is equal to Vdt, and

$$dE = \epsilon \, V \, dA \, dt. \tag{2}$$

This relation will first be used to compute $\bar{\epsilon}\bar{r}^2$. Assume that the acoustical power level of the point source is known to be

$$PWL = D \text{ db re } 10^{-13} \text{ watt} \tag{3}$$

which means that the power, P, of the source is

$$P = 10^{\frac{D-130}{10}} \text{ watt.} \tag{4}$$

* The symbol ϵ replaces the symbol ρ as used in Reference 1, since the latter will denote gas density in the present report.

While in a close neighborhood of the source, the propagation velocity can be taken as constant with height; the presence of wind will cause it to have different values depending on the azimuth, φ, at which the vertical plane under consideration is erected.

A half-sphere with the small radius $\bar{r}$ about the source can be divided (comp. Figure 1) into vertical slices of opening angle $d\varphi$ whose surfaces are

$$dA = \bar{r}^2 \, d\varphi.$$

Through each of these elements passes the energy

$$P \frac{d\varphi}{2\pi}$$

per unit time, provided that the radiation is uniform in all directions. It follows from relation (2) that

$$\frac{dE}{dt} = P \frac{d\varphi}{2\pi} = \bar{\epsilon} \, V_O \, \bar{r}^2 \, d\varphi$$

where $\bar{\epsilon}$ is the volume density of energy on the slice surface, and V_O is the propagation speed near the source. Thus,

$$\bar{r}^2 \, \bar{\epsilon} = \frac{P}{2\pi \, V_O} \, . \qquad (5)$$

The value of the product at left depends on V_O, i.e., on the azimuthal direction of the vertical plane selected. Extended sources can be considered as point sources if far-field effects are of interest only; however, their radiation might not be isotropic. Relation (5) then offers an approximation only. In severe cases one would have to resort to measurements of the energy radiated in the different directions, the total amount being of no use in these circumstances.

From relation (2) the intensity, i.e., the energy passing per unit time through the unit frontal area, is seen to be

$$I = \epsilon V. \tag{6}$$

When the radiation arrives at the ground again, $\epsilon = \epsilon_s$, $V = V_0$; expression (1) may then be written as

$$I_s = \frac{P}{2\pi} \frac{1}{x_s} \left|\frac{d\theta_0}{dx_s}\right| \text{ cotg } \theta_0 \frac{\text{watts}}{\text{length}^2} . \tag{7}$$

The reference level for intensities is variously given in the literature, either as

$$I_1^* = 10^{-13} \frac{\text{watts}}{(\text{ft})^2} \text{ , or as}$$

$$I_2^* = 10^{-16} \frac{\text{watts}}{\text{cm}^2} . \tag{8}$$

Since $1(\text{ft})^2 \approx 9.29 \times 10^2 \text{ cm}^2$, these values are nearly equal. They correspond roughly to the minimum intensity required for a 1000 cps sound excitation to be registered by the average person.

Intensity level in db is defined as 10 times the Briggs' logarithm of the ratio I_s/I^*. Since

$$\log_{10} \frac{1}{2\pi} \approx -0.8,$$

one obtains from the expression (7) and the two conventions (8), that

$$IL = \left[D - 8 + 10 \log_{10} \frac{1}{x_s} \left|\frac{d\theta_0}{dx_s}\right| \text{ cotg } \theta_0\right] \text{ db re } 10^{-13} \frac{\text{watts}}{\text{ft}^2}$$

$$IL = \left[D + 22 + 10 \log_{10} \frac{1}{x_s} \left|\frac{d\theta_0}{dx_s}\right| \text{ cotg } \theta_0\right] \text{ db re } 10^{-16} \frac{\text{watts}}{\text{cm}^2}. \tag{9}$$

The numerical value of D is connected with that of the source's acoustical power by formula (4) or (3). The argument of the logarithm, for a piecewise linear velocity distribution depending on height alone, can be obtained from the expressions (31) and (33) derived in Reference 1. For correct computation of the intensity level it is necessary to express the landing distance, x_s, which originally may be given in any arbitrary length unit, in terms of ft or cm, respectively.

For example, if the propagation velocity is a linear and increasing function of height, the argument of the logarithm for small initial ray elevations θ_0 is roughly $1/x_s{}^2$. At x_s = 3 km distance the two expressions (9) then give

$$IL = [D - 87.87] \text{ db re } 10^{-13} \frac{\text{watts}}{(\text{ft})^2}$$

$$IL = [D - 87.54] \text{ db re } 10^{-16} \frac{\text{watts}}{\text{cm}^2} .$$

The results are nearly equal, because the reference intensities are almost equal. These figures, as was mentioned before, do not allow for the effects of physical attenuation through molecular absorption, turbulent scattering and the like.

For possible practical applications, it may be mentioned that, in Reference 2, the acoustical power level for a Saturn booster delivering 1.5×10^6 pounds of thrust has been estimated as

$$PWL = 209 \text{ db re } 10^{-13} \text{ watt},$$

so that D = 209 in this case. The exhaust velocity was taken as 8500 ft/sec, and it was assumed that one percent of the jet's total power is converted into acoustical power. This figure is not very definite; for example, 1/2% is sometimes quoted in the literature for static tests of Saturn and Jupiter. Then, D = 202.

II. FIRST APPROXIMATION LINKS IN A SOUND PULSE

In experimental practice, sound pressure levels rather than intensity levels are measured. For comparison with observational evidence, it appears necessary to set up a theoretical relationship of the average intensity (9) and the average sound pressure existing in a pulse. A first order approximation to this relation has been obtained by Blokhintzev [3], although in a somewhat cursory manner; the derivation furthermore uses assumptions made elsewhere in the treatise and not mentioned again. A fuller account of the way leading to the relationship is desirable, and is given here. The notation used is that of Reference 1 (excepting energy density which is now called ϵ).

The state of the undisturbed atmosphere (including the wind velocity vector) is allowed to change from spot to spot, but not so in time; it is steady in the aerodynamic sense. An upper index (o) will be used to characterize it.

A sound disturbance wandering across the stationary atmosphere will introduce a time dependency in such oscillatory quantities as velocity ($\underline{w}$), pressure (p), density (ρ), specific entropy (s); the latter replaces the customary quantity of temperature. "Sound pressure" as observed will be the root mean square over the oscillation time:

$$p_m = \sqrt{\overline{p^2}} \;. \tag{10}$$

This is the quantity which is to be related to the energy density. For plane or point sources in a windless and constant-state atmosphere, this can be achieved by solving the wave equation (comp., e.g., Ref. 4). However, the mathematical difficulties are all but insurmountable in more complex (and more real) cases since then the standard form of the wave equation cannot be obtained from the aerodynamic equations. An approximation procedure must be invented before one can hope for results.

The state of the inhomogeneous and anisotropic atmosphere described above may be written as

$$\begin{aligned} \underline{\tilde{w}} &= \underline{w}^{(o)} + \underline{w} \qquad (\underline{w}^{(o)} \text{ being the wind vector}) \\ \tilde{p} &= p^{(o)} + p \\ \tilde{\rho} &= \rho^{(o)} + \rho \\ \tilde{s} &= s^{(o)} + s. \end{aligned} \tag{11}$$

The wave is considered as creating an oscillatory perturbation in terms so small that their squares and products can be neglected (linear acoustics). A solution is sought of the time-dependent "local" aerodynamic equations describing the pulse motion as existing within a small neighborhood of space. In it, the oscillatory quantities can be taken as being in phase for nearly plane waves, and even with spherical waves if the wave number is large; this result issues from the rigorous treatment of such waves (comp., e.g., Ref. 4). If one takes this as true for general wave forms, the oscillatory quantities may be written as

$$\underline{w} = \hat{\underline{w}}\, e^{i\phi}, \quad p = \hat{p}\, e^{i\phi}, \tag{12}$$

etc., with the same oscillation argument

$$\phi = \omega t - k_o W$$

applying to all of them. A representative circular frequency (ω) and wave number (k_o) are thus introduced; W stands for a spatial coordinate (which will be taken in the direction of the longitudinal oscillation at the given spot). The wave number k_o is considered large (rapid oscillations). This renders the approximation less trustworthy for long-wave sound for which, however, ray acoustics are of doubtful value anyway since diffraction effects caused by atmospheric turbulence may become noticeable if not dominant. On the other hand, with k_o large, the amplitudes may be expanded in terms of $1/ik_o$, which step is fundamental in Blokintzev's approach to the problem:*

$$\hat{\underline{w}} = \hat{\underline{w}}' + \frac{1}{ik_o}\hat{\underline{w}}'' + \ldots, \quad \hat{p} = \hat{p}' + \frac{1}{ik_o}\hat{p}'' + \ldots, \text{ etc.}$$

* An expansion of this kind is not feasible if, in expressions (12), the real rather than the complex representation is chosen. The indispensable relation (13) can then be obtained by making use of the local character of the solution: The oscillation amplitudes, their change in time and space, and the local change of the "undisturbed" parameters must all be considered small. It is found again that the first approximation (13) should improve if k_o is large.

Terms with n primes are called here the n^{th} approximation of the amplitude in question (by B. the $(n-1)^{th}$). For the determination of these approximations, the aerodynamic equations are available; one finds that they can be divided through by the oscillation term $e^{i\emptyset}$. It can be shown that the quantities ω and k_o have no bearing at all on ray patterns and energy contents and that, if instead of the simple oscillations (12) a sum or series of superimposing linear oscillations is prescribed, the equations for the amplitude approximations remain unchanged. They are obtained as a group of five for each approximation, so that the corresponding amplitudes of the velocity vector, of the pressure and the entropy can be determined. (By the equation of state, taken as $\tilde{p} = R\tilde{\rho}\ \tilde{T}(\tilde{\rho}, \tilde{s})$, the density has been removed as an explicit quantity.)

Regarding the first approximation, it is found that $\hat{s}' = 0$ and that the remaining amplitudes are governed by the equations

$$\underline{\hat{w}}' q - \frac{\hat{p}'}{\rho^{(o)}} \text{grad } W = 0$$

$$q \frac{\hat{p}'}{c^{(o)2}} - \rho^{(o)} \underline{\hat{w}}' \cdot \text{grad } W = 0$$

where $c^{(o)}$ is the thermodynamic sound speed at the location where the oscillation takes place at the moment. This system of four algebraic equations is homogeneous so that one of the unknowns, e.g., the first approximation pressure amplitude $\hat{p}'$, may be prescribed at will. It must and can be given an approximate definitive value only if the power of the source is known and if a relationship of the intensity level (9) (which contains the power in the quantity D) and the sound pressure level is established. The above homogeneous system has solutions different from zero on condition that its determinant is zero, leading to the requirement

$$q = c^{(o)} \ |\text{grad } W|.$$

It is seen that the meaning of the quantity q is of no interest in the present context; it can be removed, for example, from the first equation, to give

$$\underline{\hat{w}}' = \underline{n} \ \frac{\hat{p}'}{\rho^{(o)} \ c^{(o)}} \tag{13}$$

where $\underline{n}$ is the unit vector in the direction of grad W, i.e., in the direction of the wave front normal. This relation will prove to be material in setting up the desired connection of ϵ and p_m. It shows also that the first approximation to the oscillation velocity is normal to the wave front at the spot considered, as it ought to be with longitudinal oscillations.

An equation connecting vibrational pressure and density is also needed. Taking entropy and density as the independent variables, the oscillatory pressure may be expanded into a Taylor series

$$p = \left(\frac{\partial p}{\partial \rho}\right)_{\substack{\rho=o\\ s=o}} \rho + \left(\frac{\partial p}{\partial s}\right)_{\substack{\rho=o\\ s=o}} s + \ldots$$

which, in linear acoustics, may be truncated after the first two terms. The partial derivatives can be obtained from the equation of state; on replacing temperature by specific entropy, it assumes the form

$$\tilde{p} = \text{const.}\ \tilde{\rho}^{\gamma}\, e^{\frac{\tilde{s}}{c_v}} \quad \text{with } \gamma = \frac{c_p}{c_v}.$$

The undisturbed state of the atmosphere is independent of ρ and s, so that logarithmic partial derivation with respect to ρ and s gives

$$\frac{1}{\tilde{p}}\frac{\partial p}{\partial \rho} = \gamma\,\frac{1}{\tilde{\rho}}\,, \quad \frac{1}{\tilde{p}}\frac{\partial p}{\partial s} = \frac{1}{c_v}\,.$$

On applying the condition $\rho = 0$, $s = 0$ one finds that

$$p = c^{(o)^2}\rho + \frac{p^{(o)}}{c_v}\, s \tag{14}$$

as

$$\gamma\,\frac{p^{(o)}}{\rho^{(o)}} = c^{(o)^2}\,.$$

This equation holds for the amplitudes $\hat{p}$ etc., as well, since the common factor $e^{i\phi}$ cancels out.* In the first approximation ($\hat{s}' = 0$) we have

$$\hat{p}' = c^{(o)^2} \hat{\rho}' . \tag{15}$$

III. LOCAL RELATIONSHIP OF INTENSITY LEVEL AND SOUND PRESSURE LEVEL

The kinetic energy contained in a pulse is obtained by subtracting the drift or "undisturbed" energy from the total kinetic energy; its volume density is therefore

$$\kappa = \frac{1}{2} \tilde{\rho} \, \underline{\tilde{w}}^2 - \frac{1}{2} \rho^{(o)} \, \underline{w}^{(o)^2} .$$

On using the approaches (11), this gives

$$2\kappa = \rho^{(o)} \, (2\underline{w}^{(o)} \cdot \underline{w} + \underline{w}^2) + \rho \, (\underline{w}^{(o)^2} + 2\underline{w}^{(o)} \cdot \underline{w} + \underline{w}^2).$$

For determining the average of κ over the oscillation period, T, one best changes the complex into the real representation employing, e.g., the cosine function

$$e^{i\phi} \to \cos(\phi + \text{phase term}).$$

As the integrals over the odd powers of the cosine function vanish and that over the square is T/2,

$$2\bar{\kappa} = \frac{1}{T} \int_0^T 2\kappa \, dt = \frac{\rho^{(o)} \, \underline{\hat{w}}^2}{2} + \hat{\rho} \, \underline{w}^{(o)} \cdot \underline{\hat{w}} .$$

---

* So does the oscillatory term in the real representation.

The second term in this expression is clearly caused by the presence of the wind ($\underline{w}^{(o)}$), which takes the oscillation along. The first term is twice the average of the specific kinetic energy contained in the vibration itself and is therefore, in a linear oscillation, equal to twice the time average of the specific potential energy existing in the pulse. The latter must be added to obtain the volume density of the total vibrational energy which thus emerges as

$$\epsilon = \frac{1}{2} (\rho^{(o)} \underline{\hat{w}}^2 + \hat{\rho}\, \underline{w}^{(o)} \cdot \underline{\hat{w}}). \tag{16}$$

If for the quantities $\hat{\rho}$ and $\underline{\hat{w}}$ their first approximations are introduced, they may be replaced by $\hat{p}'$ with the aid of the relations (13) and (15). Since

$$\frac{1}{2} \hat{p}'^2 \approx \frac{1}{2} \hat{p}^2 = \overline{p^2} = p_m{}^2,$$

one finds that

$$\epsilon = p_m{}^2 \frac{c^{(o)} + \underline{n} \cdot \underline{w}^{(o)}}{\rho^{(o)}\, c^{(o)3}} . \tag{17}$$

The propagation velocity of the energy being

$$\underline{V}_s = \underline{n}\, c^{(o)} + \underline{w}^{(o)},$$

the expression (6) for the intensity then gives

$$I = p_m{}^2 \frac{c^{(o)} + \underline{n} \cdot \underline{w}^{(o)}}{\rho^{(o)}\, c^{(o)3}} |\underline{V}_s|. \tag{18}$$

Formula (18) is the desired relation between the average intensity and the measured sound pressure in a first approximation. Upper indices (o) refer to the state of the atmosphere at the location occupied by the pulse at a given moment. As a rule, these quantities are functions of space (but not of time in a steady atmosphere). If the square of the wind velocity is small when compared to the square of the thermodynamic sound speed (as common practice assumes), it can be shown [1] that

$$|\underline{V}_s| \approx c^{(o)} + n \cdot \underline{w}^{(o)} = V_f$$

where V_f is the speed with which the wave front moves in direction of its local normal (wave speed). With this simplification,

$$I = p_m^2 \frac{V_f^2}{\rho^{(o)} c^{(o)3}} . \tag{19}$$

In a windless atmosphere $[\underline{w}^{(o)} = 0]$, expression (18) assumes the familiar form

$$I = \frac{p_m^2}{\rho^{(o)} c^{(o)}} , \tag{20}$$

which rule comes out here as a first approximation result, but, for plane and spherical waves, can be derived from the rigorous solution of the wave equation; see, e.g., Reference 4. It appears now that, as a first approximation, this relation has a much wider scope. It is not confined to special types of waves that can exist in a homogeneous atmosphere ($\rho^{(o)} c^{(o)}$ = const.); in fact, the only restriction made is the absence of wind.

As an immediate application, we may determine the value of the acoustical impedance, $\rho^* c^*$, related to the reference pressure

$$p^* = 0.0002 \ \mu b \tag{21}$$

and either one of the two values given for the reference intensity I^* in formula (8). One finds that

$$\begin{aligned} (\rho^* c^*)_1 &= 37.16 \ \frac{\text{dyne sec}}{\text{cm}^3} \qquad \text{for } I_1^* = 10^{-13} \ \frac{\text{watts}}{(\text{ft})^2} \\ (\rho^* c^*)_2 &= 40 \ \frac{\text{dyne sec}}{\text{cm}^3} \qquad \text{for } I_2^* = 10^{-16} \ \frac{\text{watts}}{\text{cm}^2} . \end{aligned} \tag{22}$$

These figures may be compared to those holding in a homogeneous atmosphere at sea level condition everywhere. Reference 5 gives, on p. 222, that at sea level

$$c^{(o)} = 344 \text{ m/sec}, \quad \rho^{(o)} = 0.00121 \text{ g/cm}^3;$$

therefore,

$$c^{(o)} \rho^{(o)} = 41.62 \frac{\text{dyne sec}}{\text{cm}^3} .$$

This agrees with the findings from the model atmosphere published in Reference 6, where we learn that, at sea level,

$$T^{(o)} = 288.16°\text{K} \rightarrow c^{(o)} \approx 340 \text{ m/sec, from p. 1-10}$$

$$\rho^{(o)} = 1.225 \text{ kg/m}^3\text{, from p. 1-13, so that again}$$

$$c^{(o)} \rho^{(o)} = 41.6 \frac{\text{dyne sec}}{\text{cm}^2} .$$

In such a sea level atmosphere the intensity level is numerically nearly equal to the sound pressure level. Indeed, the equation holds [by (20)]:

$$10 \log_{10} \frac{I}{I^*} + 10 \log_{10} \frac{\rho^{(o)} c^{(o)}}{\rho^* c^*} = 20 \log_{10} \frac{p_m}{p^*}$$

and the second term at left is found as 0.49 db and 0.17 db with the reference values I_1^* and I_2^*. With high levels, small differences like these might be considered as negligibly small, and the sound pressure level may be taken as equal to the intensity level. However, in the real atmosphere, the acoustical impedance may markedly differ from $\rho^* c^*$. The difference may then amount to several decibels.

For example, consider the atmosphere as two-dimensional as defined in Reference 1. Assume furthermore that the ray inclination is always small and that the wind is horizontal, so that the expression for the wave (and energy propagation) speed can be written in the simple form

$$V_f = c^{(o)} + u_1$$

where u_1 is the component of the horizontal wind in the azimuthal half-plane to which the sound ray pattern is confined in the two-dimensional analysis. Expression (19) which is pertinent here can now be set into the form

$$10 \log_{10} \frac{I}{I^*} + \log_{10} \frac{\rho^{(o)} c^{(o)}}{\rho^* c^* \left(1 + \frac{u_1}{c^{(o)}}\right)^2} = 20 \log_{10} \frac{p_m}{p^*} \tag{23}$$

From the model atmosphere given in Reference 6 it is seen that, at 4 km height, for example,

$$\rho^{(o)} = 0.819 \text{ kg/m}^3, \; T^{(o)} = 262.18°\text{K} \rightarrow c^{(o)} \approx 324 \text{ m/sec}$$

$$\rho^{(o)} c^{(o)} \approx 26.54 \frac{\text{dyne sec}}{\text{cm}^3} \quad .$$

If we take u_1 as 10 percent of $c^{(o)}$, the second term at left assumes the values

- 2.3 db and - 2.6 db

for the reference intensities I_1^* and I_2^*. By so much is the intensity level higher here than the sound pressure level.

In passing it may be noted that I_1^* will always produce a difference that is by ≈ 0.32 db algebraically larger than that caused by I_2^*, since

$$10 \log_{10} \frac{40}{37.16} \approx 0.32.$$

The main application of relation (23) will be at the rearrival locations of rays at source level. It can then be safely assumed that the logarithmand of the second term will rarely deviate much from unity except at extremely hot, cold, and/or windy days. By and large, the impedance $\rho^{(o)} c^{(o)}$ will be close to 40, and the factor

$$\left(1 + \frac{u_1}{c^{(o)}}\right)^2$$

will be near unity. In many cases, the intensity level will be different from the sound pressure level by a fractional db-number only. However, since the computation by expression (23) of the difference is simple, one may prefer to take account of it as a matter of routine. The theoretical sound pressure level is then known somewhat more accurately than when merely equating it with the intensity level. The choice of I^* will affect the value of $\rho^* c^*$ to be used in the intensity and sound pressure level relation (23); see expressions (22).

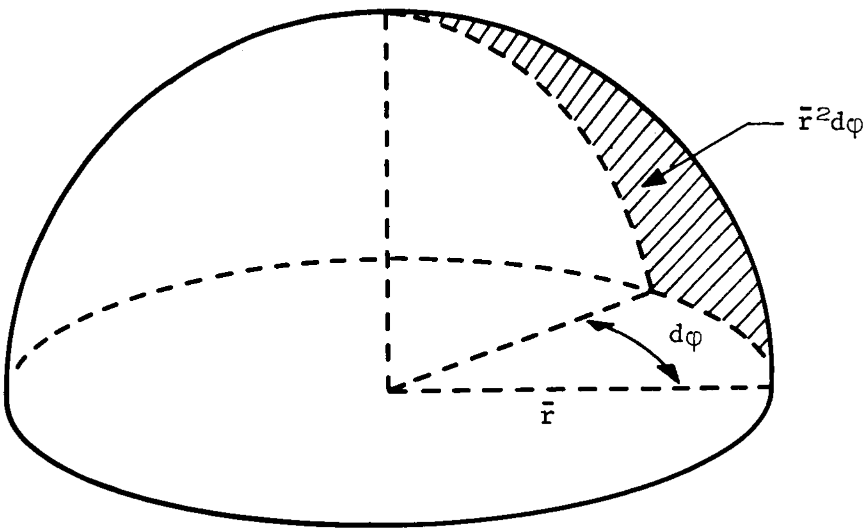

FIGURE 1. SURFACE ELEMENT OF THE HALF-SPHERE ABOUT THE SOURCE

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